



# Journée des méso-centres

16 Mai 2013, Rouen – France

## Micro-organismes dans un écoulement turbulent : effet de la gyrotaxie sur l'accumulation préférentielle

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De nombreux micro-organismes qui forment le plancton ont la faculté de se déplacer de façon autonome. Nous avons étudié l'interaction entre cette nage directionnelle (gyrotaxie) et les structures turbulentes de petite échelle que l'on trouve dans l'océan.

Ceci nous a conduit à démontrer l'existence d'un mécanisme physique original conduisant à l'accumulation préférentielle de ces micro-organismes dans des régions bien identifiées de l'écoulement. La base de notre modèle de simulation est constituée du couplage entre la résolution numérique directe des équations de Navier-Stokes 3D instationnaires et d'un suivi Lagrangien des micro-organismes. L'ensemble des simulations a été conduit sur le supercalculateur Hyperion de CALMIP.



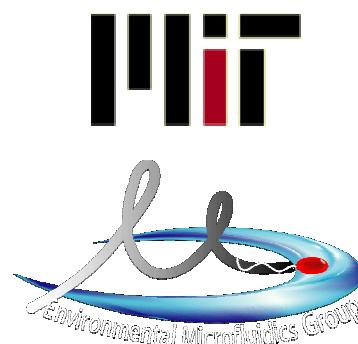
**calmip**

# Turbulent eddies and Motility generate small-scale phytoplankton patchiness



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Institute of Fluid Mechanics,  
University of Toulouse - France



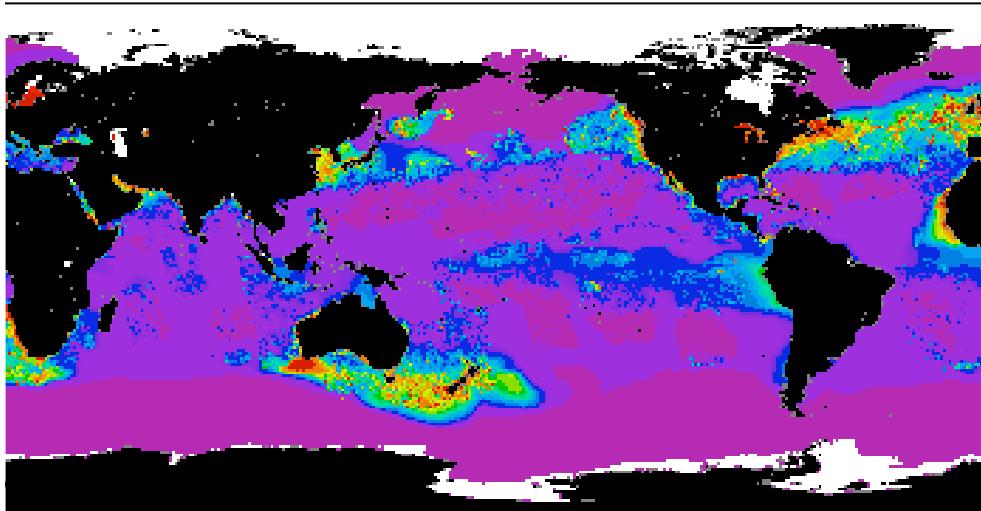
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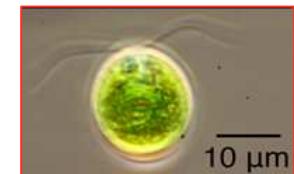
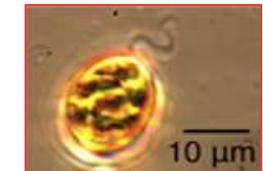
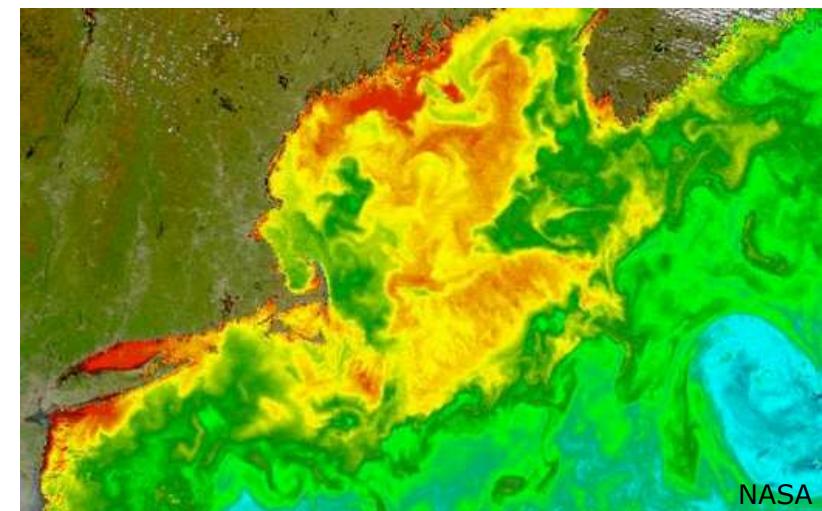
# Phytoplankton patchiness

$L > 1000 \text{ km}$



$L \approx 100 \text{ km}$

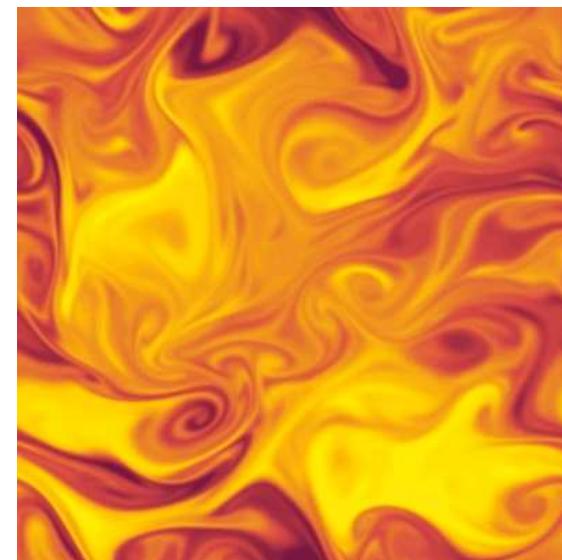
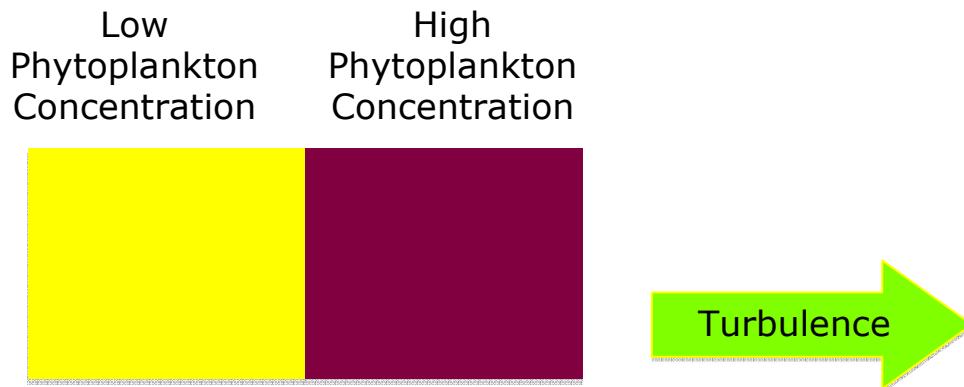
$L \approx 100 \text{ km}$



- The primary producers of the Ocean
- Form the base of the marine food web
- 50% of the global production of oxygen
- Form Harmful Algal Blooms (HABs)

# What may cause small-scale heterogeneity ?

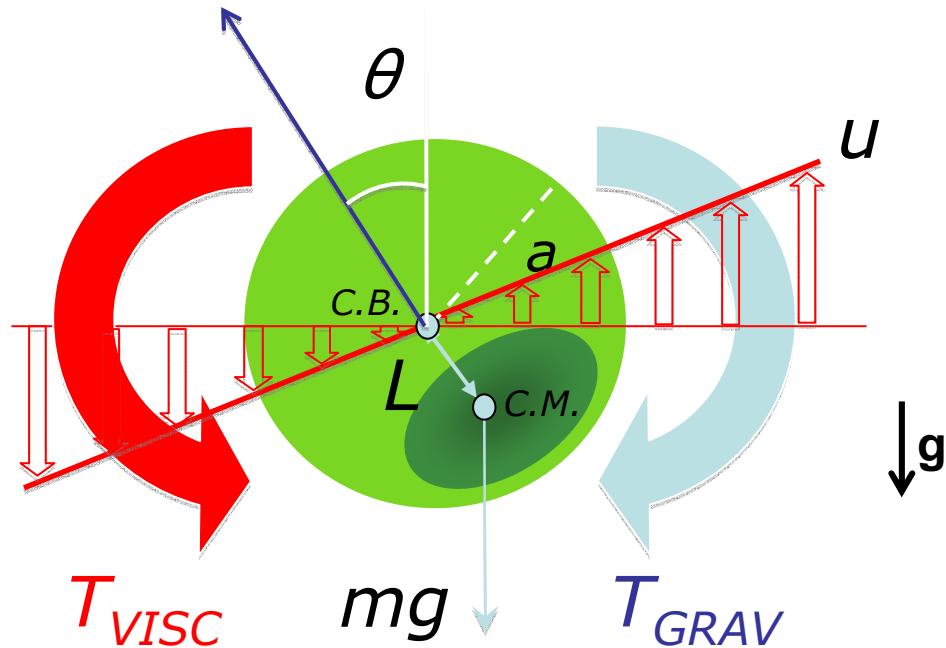
Stretching and folding of large scale heterogeneities  
by turbulent eddies    => plankton is a passive tracer of the fluid



**but the micro-organisms are motile (swimming) !!!**

# Gyrotaxis

$V_{swim} \mathbf{p}$

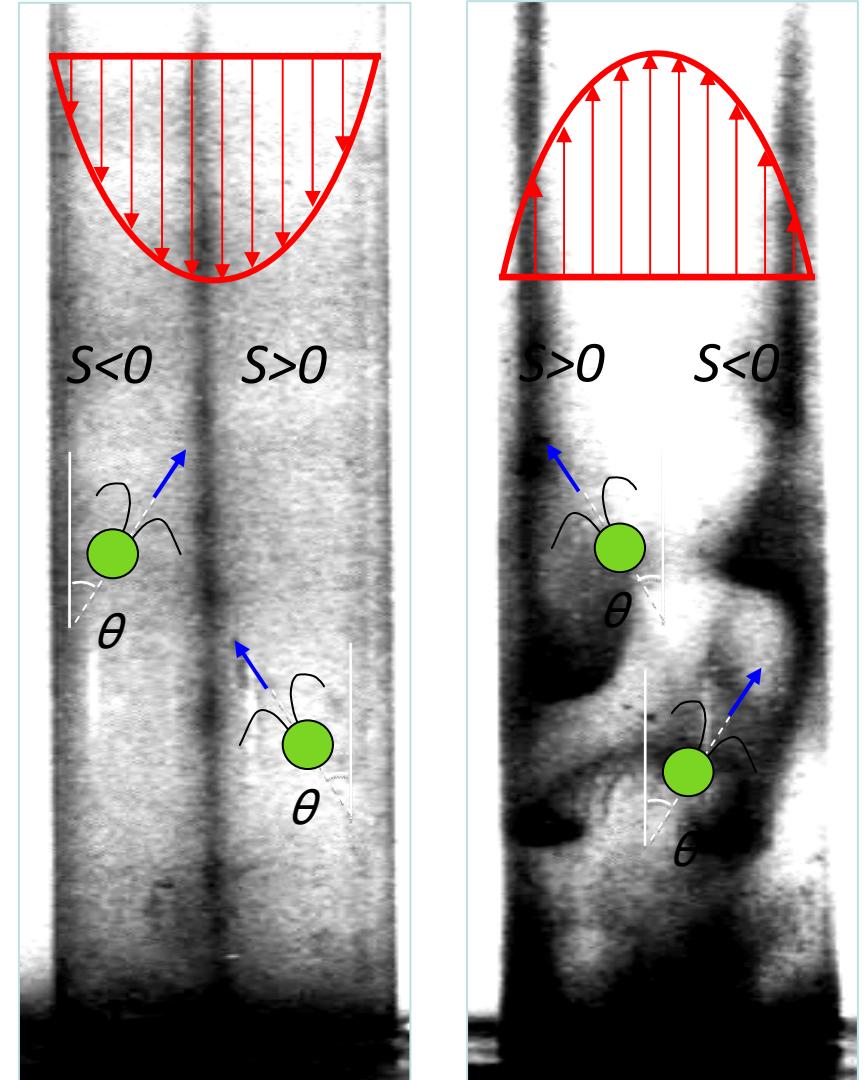


$$\frac{\partial \mathbf{p}}{\partial t} = \frac{1}{2B} [\mathbf{k} - (\mathbf{k} \cdot \mathbf{p})\mathbf{p}] + \frac{1}{2}(\boldsymbol{\omega} \times \mathbf{p})$$

Equilibrium orientation:  $T_{VISC} = T_{GRAV}$

$$\sin \theta = B \omega$$

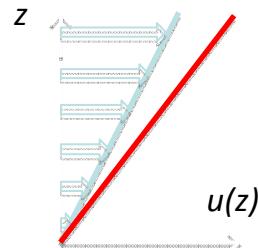
$$B = \frac{4\pi\mu a^3}{mgL}$$



(Kessler, 1985)

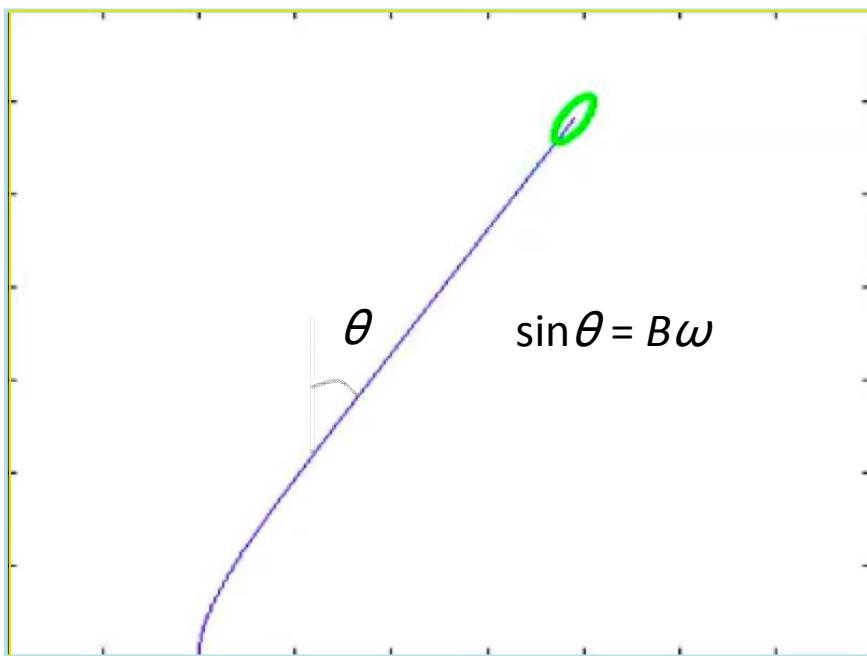
# Equilibrium regime

$$\sin\theta = B\omega$$



$$\omega < \omega_{cr} = B^{-1}$$

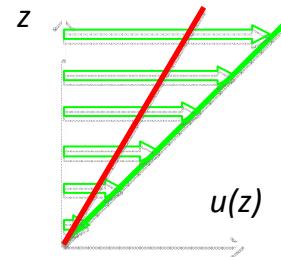
**cells swim up**



# Tumbling regime

$$\omega > \omega_{cr} = B^{-1}$$

**cells tumble**

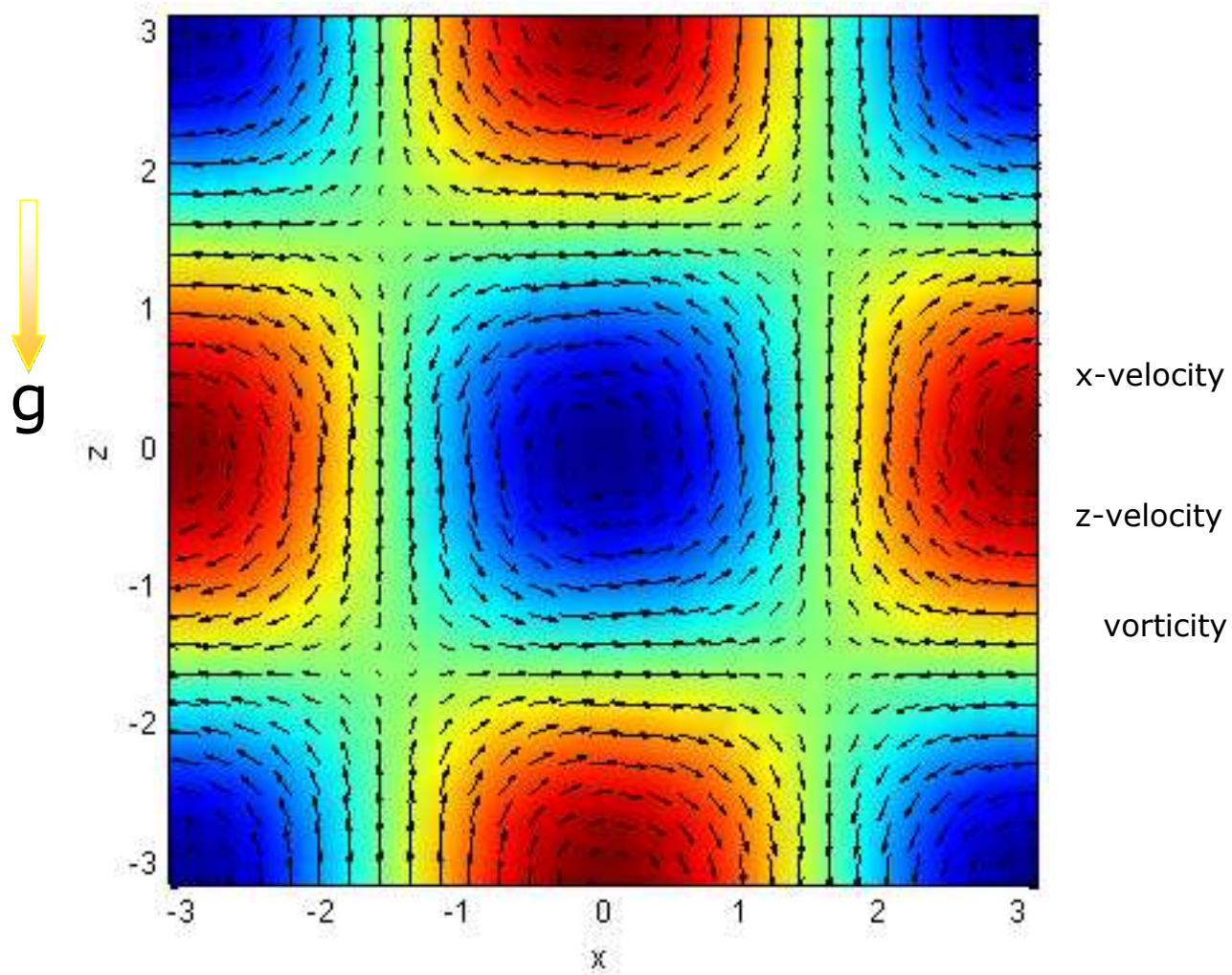


$$d\theta/dt = \gamma/2 (S - \sin\theta/B)$$

Transition occurs at  $\omega_{cr} = B^{-1}$

Durham, Kessler & Stocker, *Science* 2009

# A simple model: Taylor-Green Vortex (steady)



periodic array of 2D, alternate vortices

x-velocity

z-velocity

vorticity

$$u = -\frac{\omega_o}{2k} \cos(kx) \sin(kz)$$

$$w = \frac{\omega_o}{2k} \sin(kx) \cos(kz)$$

$$\omega = -\omega_o \cos(kx) \cos(kz)$$

# Gyrotaxis in Vortical Flow

$$\frac{\partial \mathbf{p}}{\partial t} = \frac{1}{2B} [\mathbf{k} - (\mathbf{k} \cdot \mathbf{p})\mathbf{p}] + \frac{1}{2}(\boldsymbol{\omega} \times \mathbf{p}) + \alpha_0 \mathbf{p} \cdot \mathbf{E} \cdot (\mathbf{I} - \mathbf{p}\mathbf{p})$$

**Stability imparted  
by bottom heaviness**

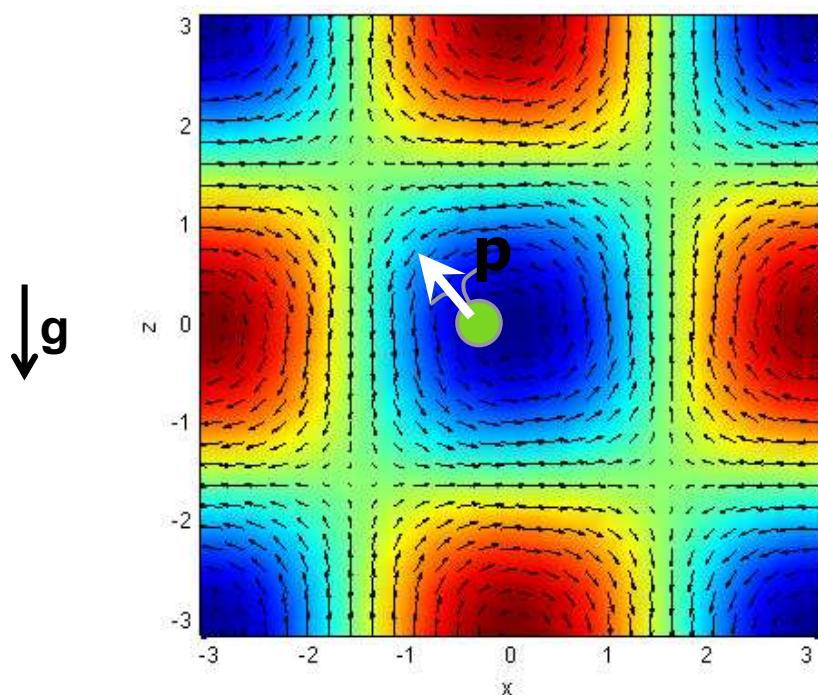
**OVERTURNING  
by vorticity**

**Alignment with  
principal strain**

Spherical cells

$\mathbf{p}$  = unit vector of swimming direction  
 $\mathbf{k}$  = unit vertical vector  
 $B$  = gyrotactic reorientation time  
 $\alpha_0$  = cell aspect ratio  
 $\mathbf{E}$  = rate of strain tensor

Leal and Hinch, 1972  
 Pedley and Kessler, 1992



Steady flow

$$\mathbf{V}(t) = \mathbf{u}(\mathbf{x}) + V_{swim} \mathbf{p}(t)$$

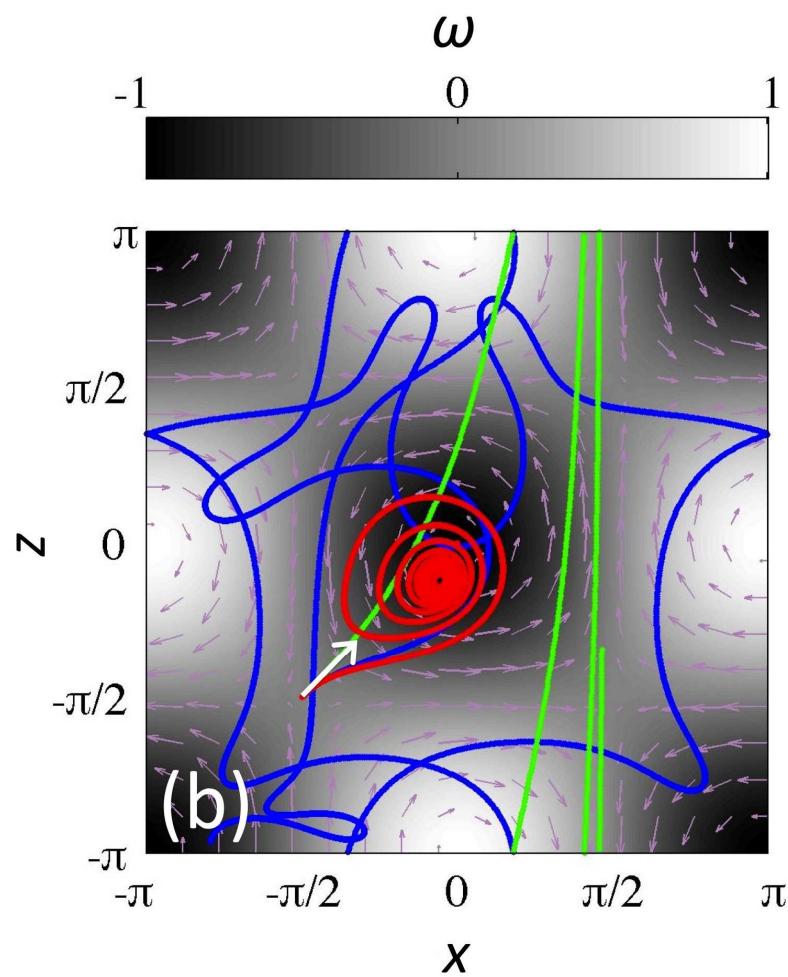
$$\frac{d\mathbf{x}}{dt} = \mathbf{V}(t)$$

Two dimensionless parameters

$$\Psi = B \omega_0$$

$$\Phi = \frac{V_C}{V_{VORTEX}} = \frac{V_C k}{\omega_0}$$

# Gyrotaxis in Vortical Flow



$$\Psi = B \omega_0$$

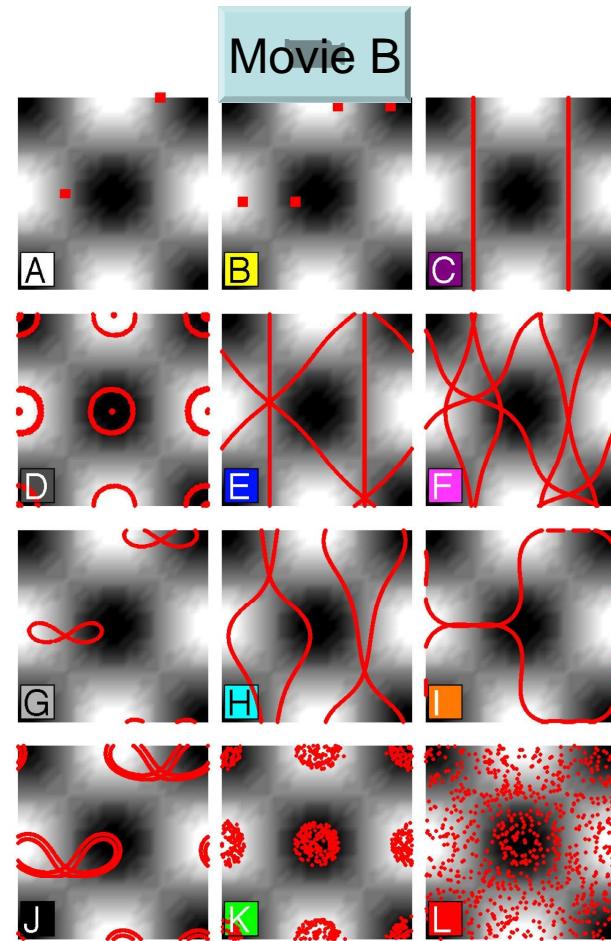
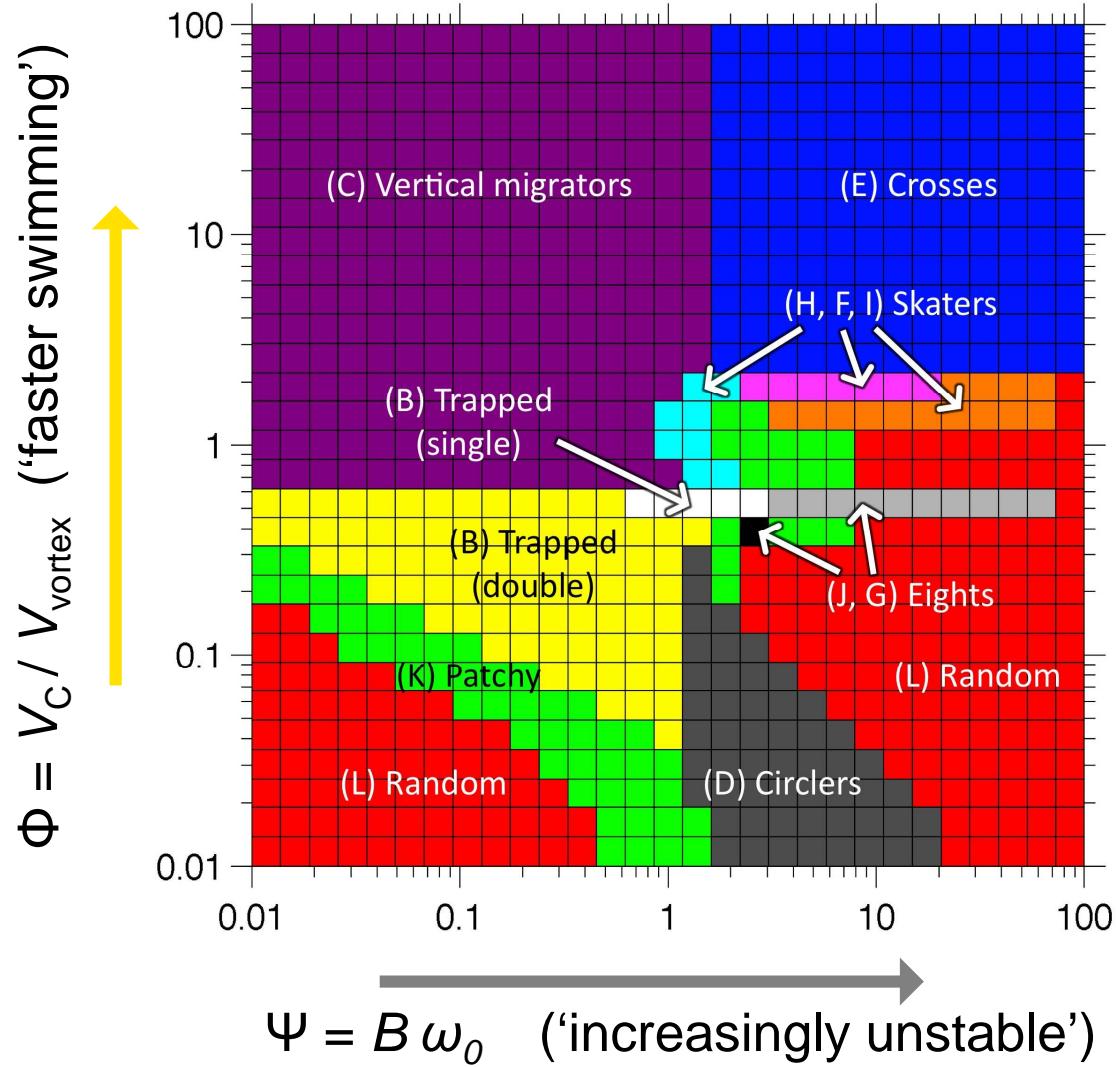
$$\Phi = \frac{V_C}{V_{VORTEX}} = \frac{V_C k}{\omega_0}$$

**g**  
↓

Three cell species (different  $\Psi$  and  $\Phi$ ) initialized at the same location and orientation ( $x = z = -\pi/2$ ;  $\theta = \pi/4$ ; white arrow)

Trajectories correspond to  
 $(\Psi=0.1, \Phi=20)$         
 $(\Psi=1, \Phi=0.2)$         
 $(\Psi=100, \Phi=0.5)$ .

# Diagram of accumulation patterns



# Description of the numerical modelling

- Continuous phase : Newtonian fluid (**Direct Numerical Simulation**)

$$\nabla \cdot \mathbf{u} = 0$$

$\rho_f, \mu$  : constant physical properties

$$\rho_f \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla P + \mu \Delta \mathbf{u} + f$$

Large scale forcing  
to sustain turbulence

- Trajectory of motile gyrotactic spherical micro-organisms

$$\mathbf{V}(t) = \mathbf{u}(\mathbf{x}, t) + V_{swim} \mathbf{p}(t)$$

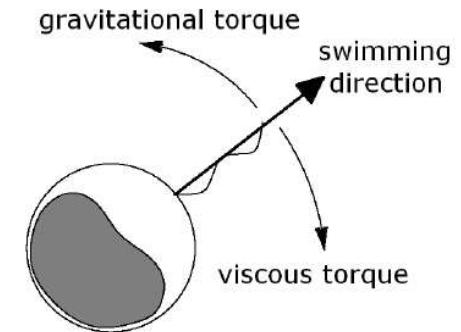
$$\frac{d\mathbf{p}}{dt} = \frac{1}{2B} (\mathbf{k} - (\mathbf{k} \cdot \mathbf{p})\mathbf{p}) + \frac{1}{2} \boldsymbol{\omega}(\mathbf{x}, t) \wedge \mathbf{p} + \alpha \mathbf{p} \cdot \mathbf{E} (\mathbf{I} - \mathbf{p}\mathbf{p})$$

**Stability imparted  
by bottom heaviness**

**OVERTURNING  
BY VORTICITY**

**ALIGNMENT WITH  
PRINCIPAL STRAIN**

Leal and Hinch, 1972  
Pedley and Kessler, 1992



$\mathbf{p}$ = unit vector of swimming direction
$\mathbf{k}$ = unit vertical vector
$B$ = reorientation time
$\alpha$ = cell aspect ratio
$\mathbf{E}$ = rate of strain tensor

# Characteristics of the flow

Homogeneous isotropic turbulence with large scale forcing  
=> sustaining turbulence and energy cascade

Fourier pseudo-spectral resolution  
=> Parallel computing using FFTW\_MPI

Tri-periodic domain



Fully resolved DNS:  $128^3$  grid points ( $k_{\max} \eta_k = 1.4$ )

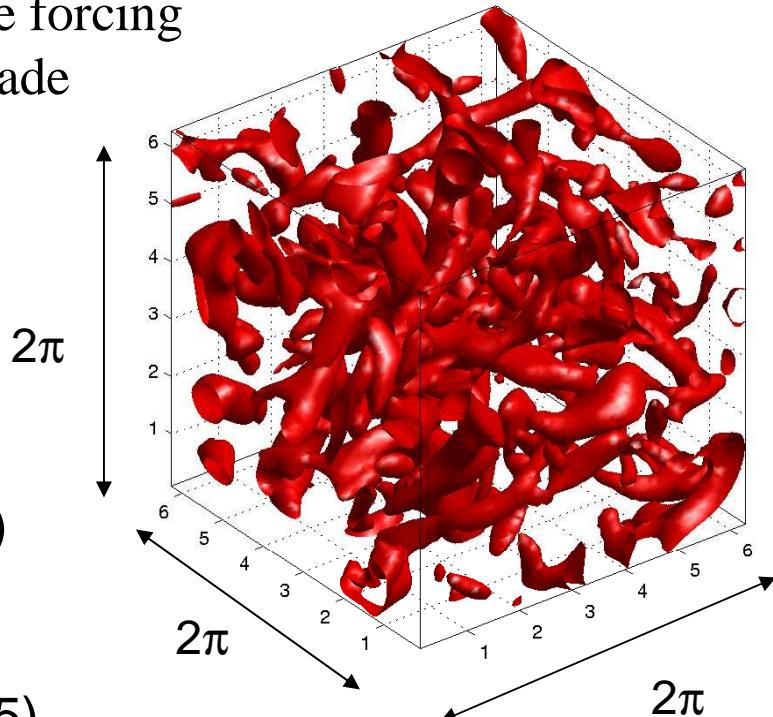
Characteristic scales of the turbulent flow ( $Re_\lambda = 55$ )

Velocity fluctuations  $u_{rms} = 19.256$

Taylor micro-scale  $\lambda = 0.3405$

Kolmogorov scales

$\tau_k = 0.0046$  ;  $v_k = 5.087$  ;  $\eta_k = 0.0234$



## Two dimensionless numbers

### Characteristics of the swimming micro-organisms

- Time scale of reorientation  $B$
- Swimming speed  $V_{\text{swim}}$

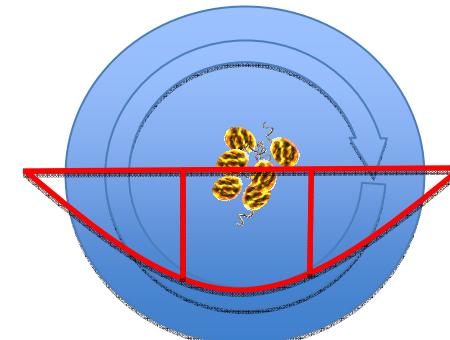
$$\Psi = B / \tau_K = B / \text{Kolmogorov time}$$

$$\Phi = \frac{V_{\text{Swim}}}{v_K} = \frac{\text{Swimming velocity}}{\text{Kolmogorov velocity}}$$

Characteristic scale of the vorticity :  $\Omega \sim (\varepsilon / 3v)^{1/2} \Rightarrow \Omega \tau_K = 1/3^{1/2}$

OVERTURNING may occur when :  $2\Omega B > 1 \Rightarrow B > 3^{1/2} \tau_K / 2 \Rightarrow \Psi > 3.5$

Gyrotactic trapping in high vorticity regions ?



# $\Phi$ and $\Psi$ in the Ocean

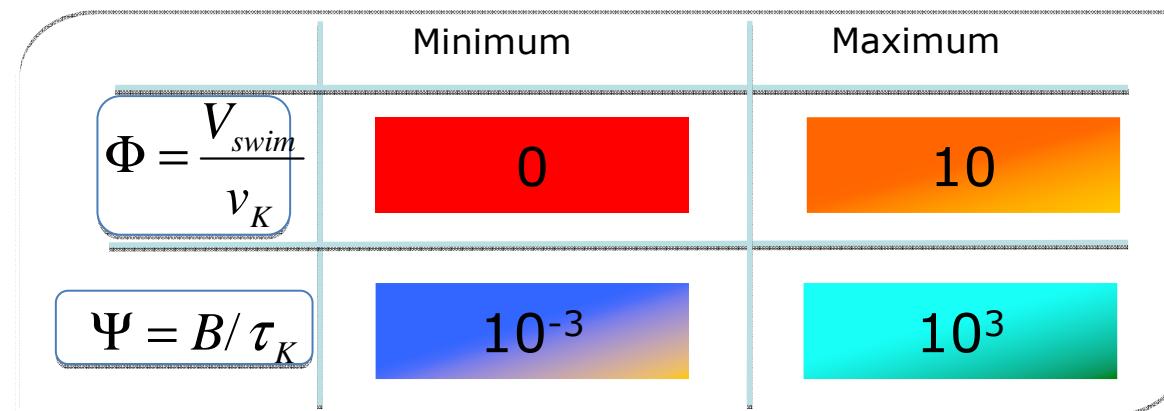
Turbulence

	$\varepsilon = 10^{-10} \text{ W/kg}$	$\varepsilon = 10^{-4} \text{ W/kg}$
$1/\tau_k = S_K = (\varepsilon/v)^{1/2}$	0.01 s <sup>-1</sup>	10 s <sup>-1</sup>
$v_K = (v\varepsilon)^{1/4}$	100 μm/s	3000 μm/s

Motility

$$B = 0.1 - 100 \text{ s} \quad (\text{only known for 3 species})$$

$$V_s = 0 - 1000 \text{ μm/s}$$



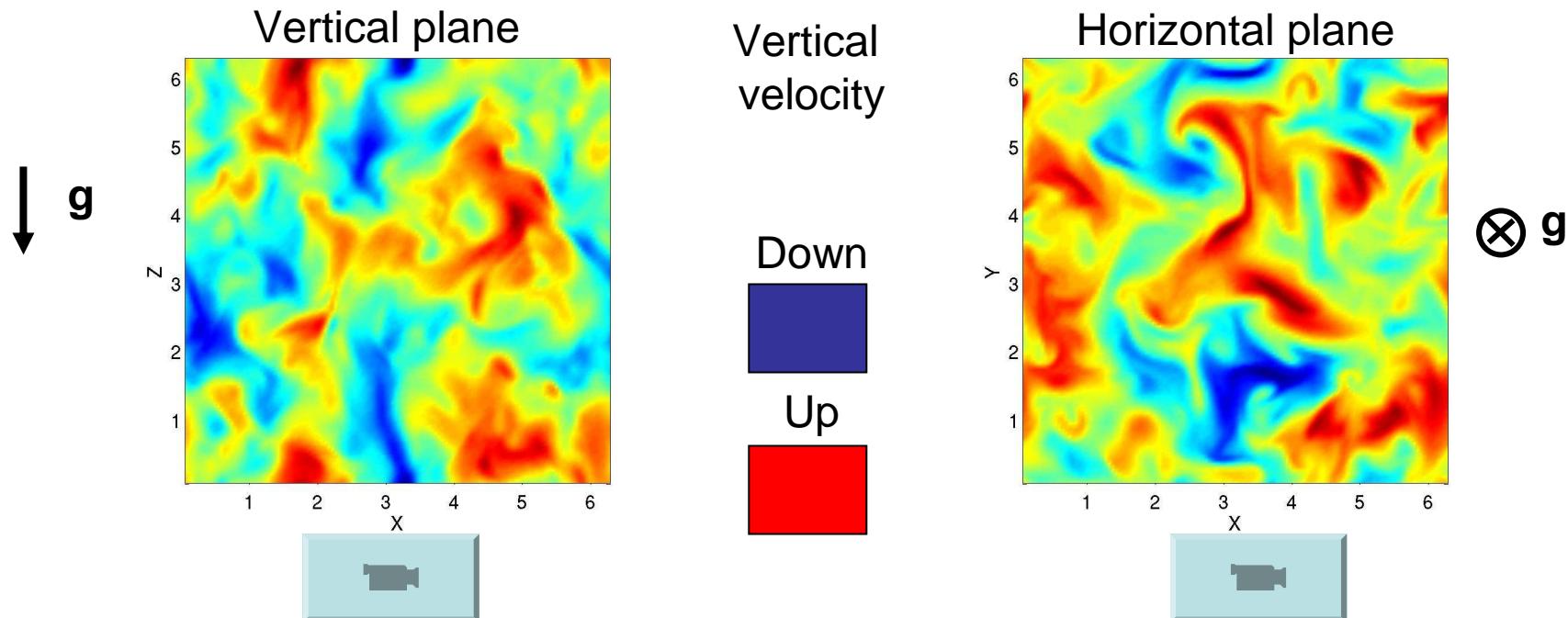
In the simulations,  $\Phi$  and  $\Psi$  were varied independently in the range :  
 $0.1 - 40$

# Accumulation does occur !

100,000 particles seeded at initial random positions

Videos over 20 Kolmogorov time scales

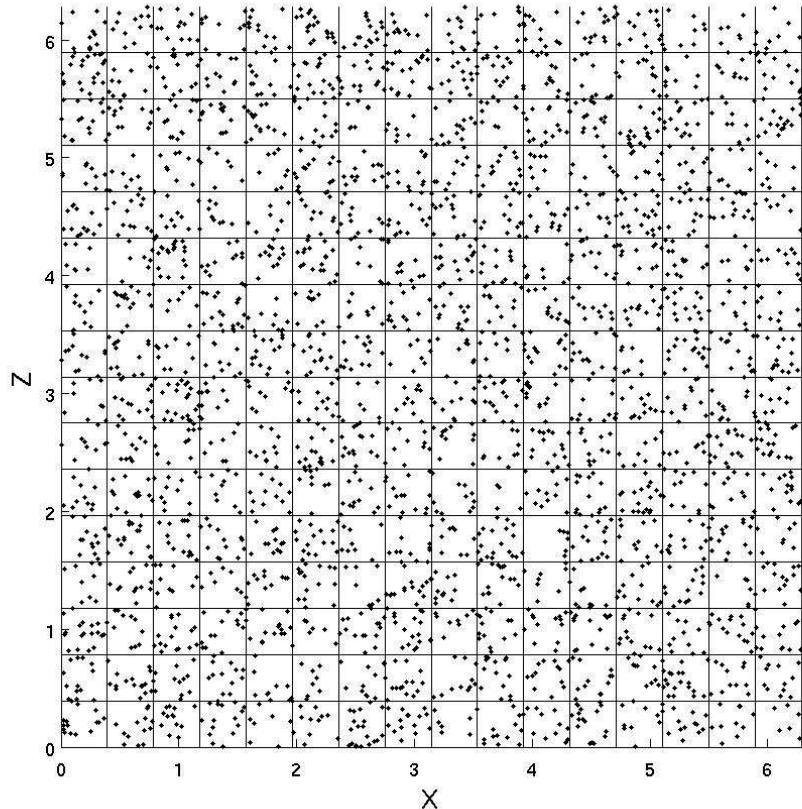
$$\Phi=10 \text{ and } \Psi=1$$



Tests with different initial random positions

Tests over the three possible orientations of gravity

# Accumulation statistics



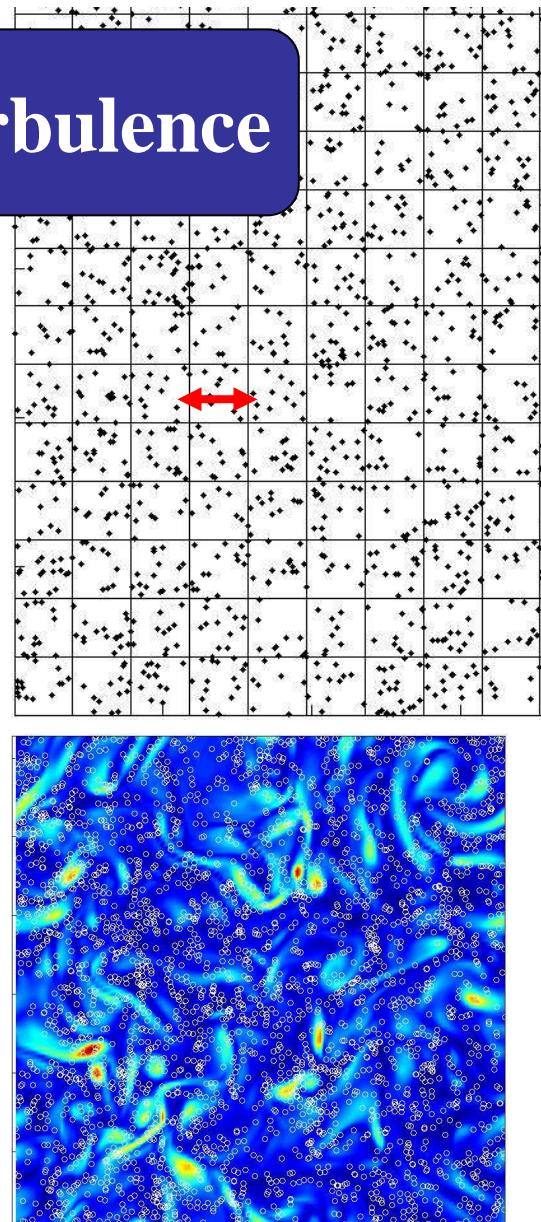
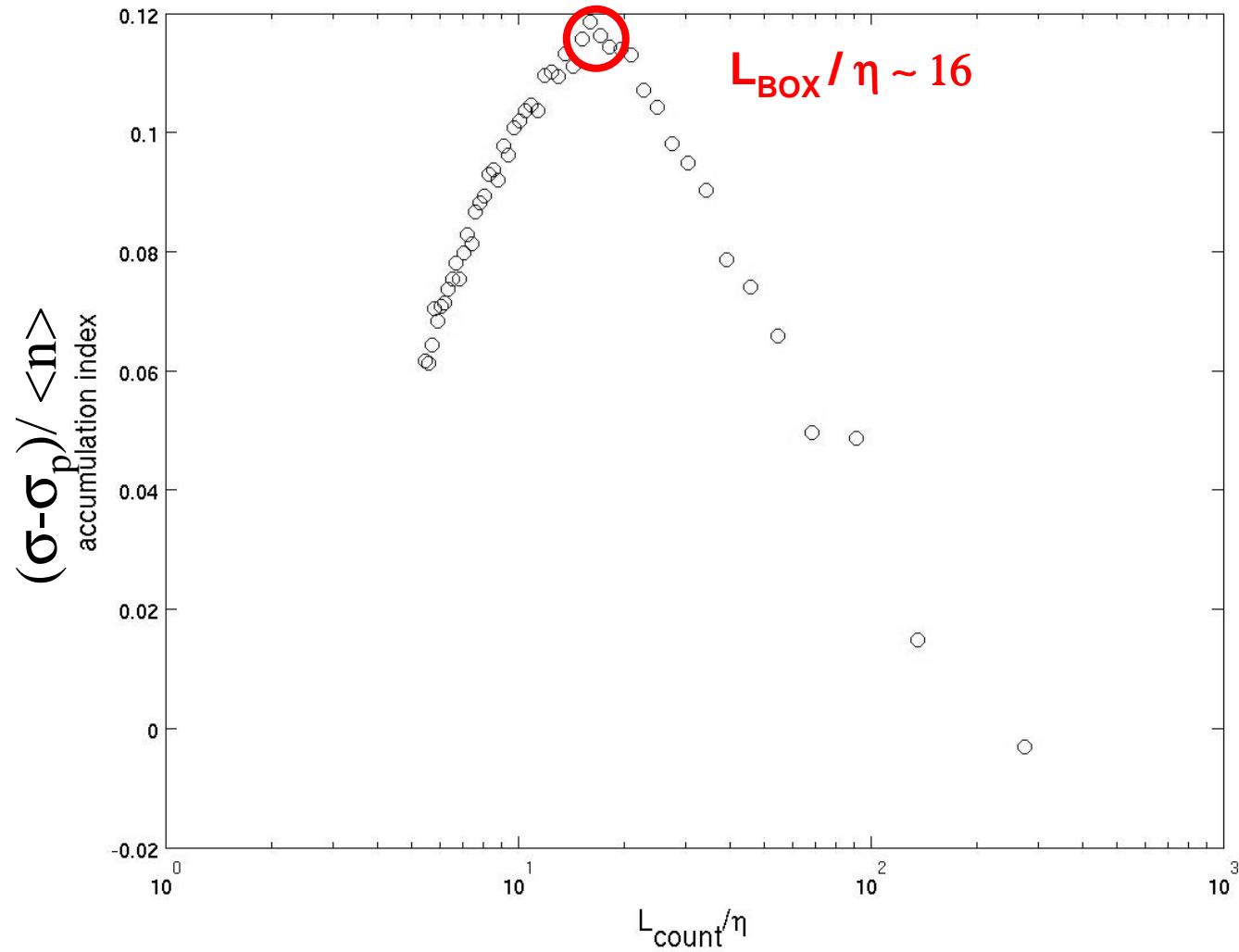
Pdf of the number  $n$  of cells in a box  
(box counting :  $L/16$ )

Random positions : Poisson distribution  
100,000 particles  $\Rightarrow \langle n \rangle = 25$   
std deviation  $\sigma_p = \langle n \rangle^{1/2} = 5$

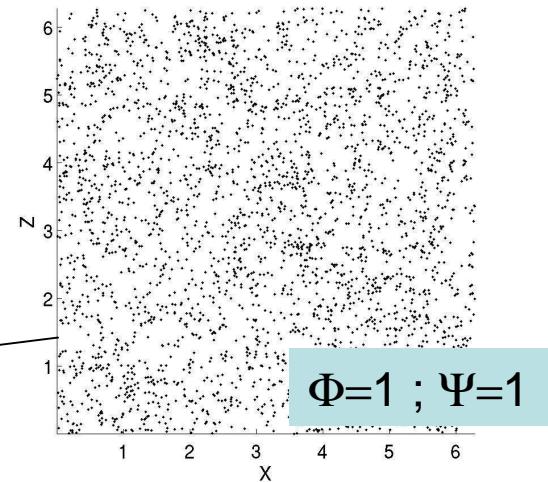
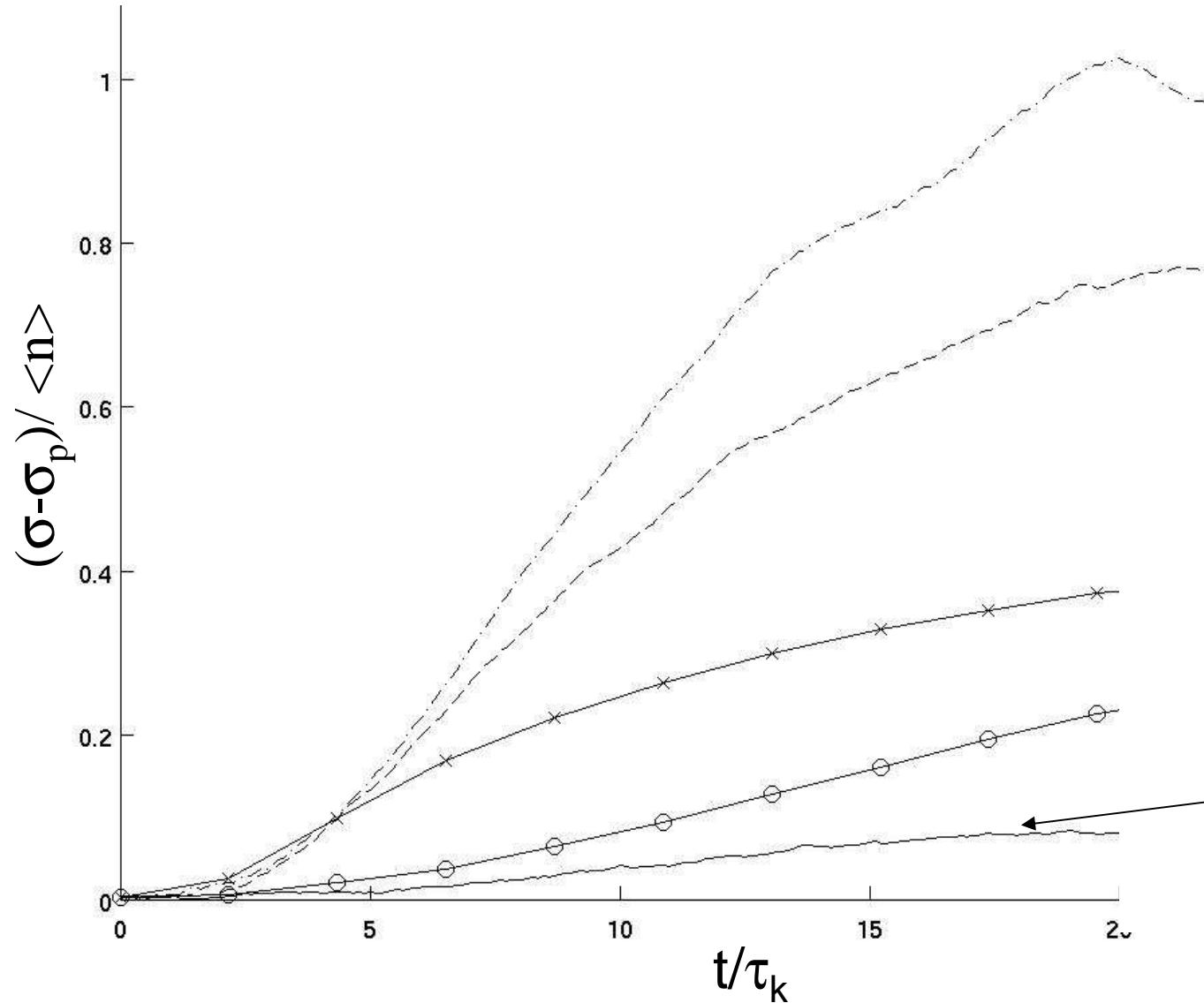
Pdf of  $n$   
Accumulation index:  $N = (\sigma - \sigma_p) / \langle n \rangle$

Voronoi tessellation

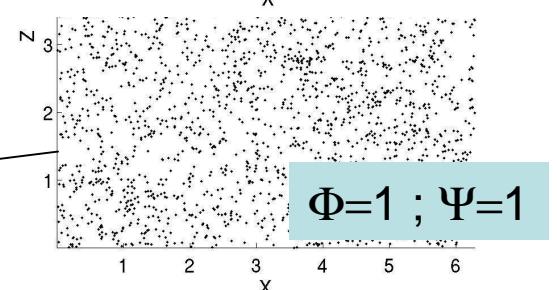
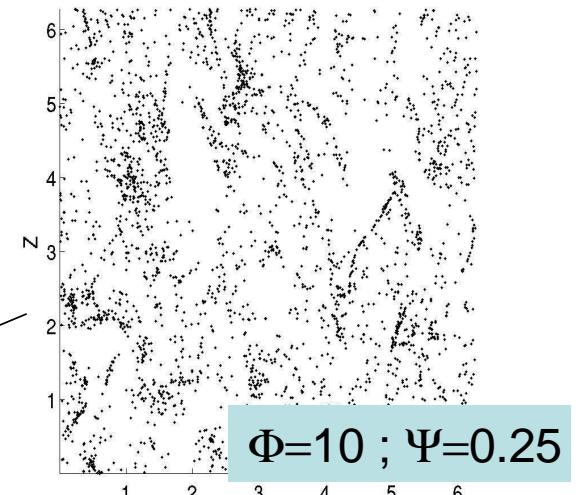
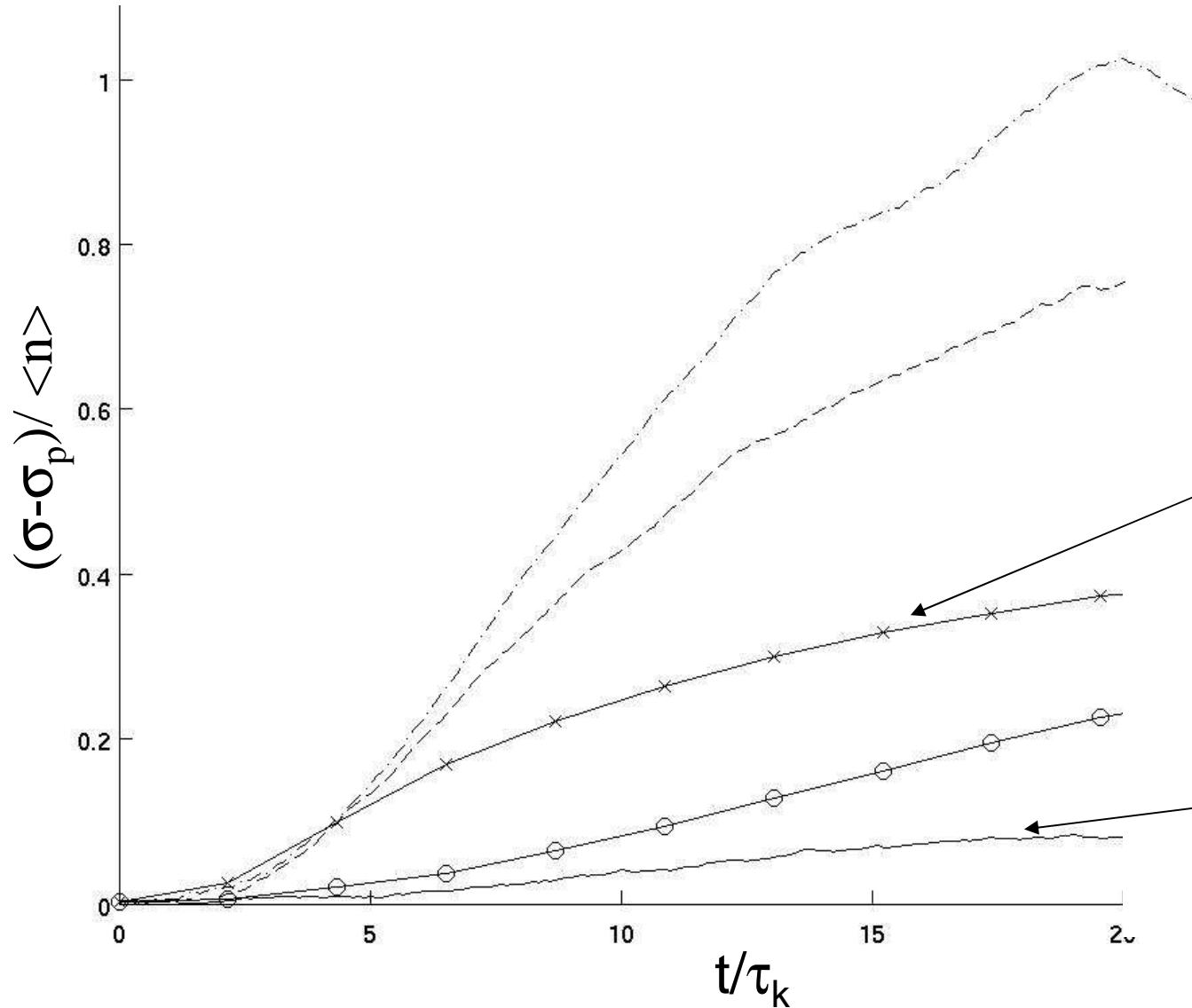
# Accumulation length scale in Turbulence



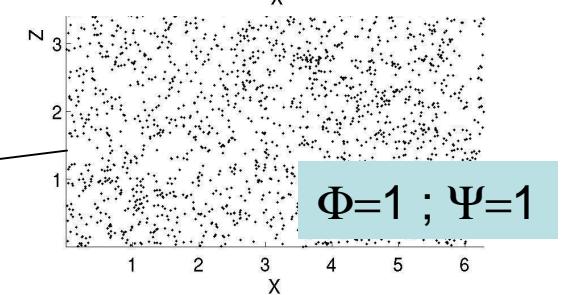
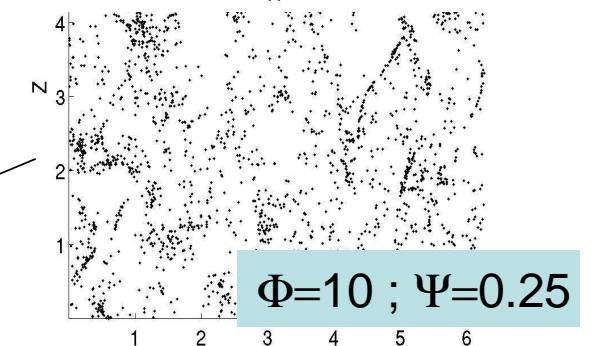
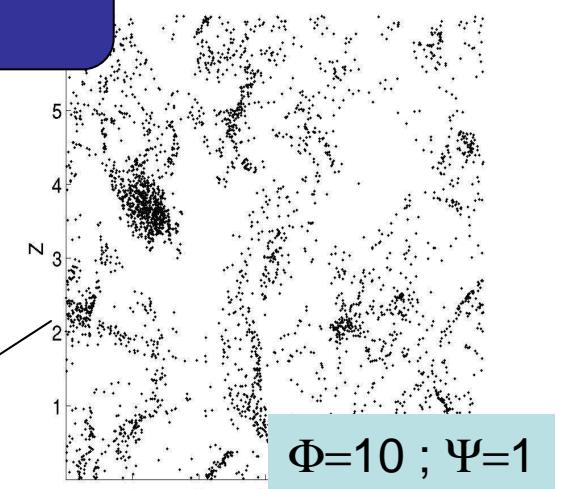
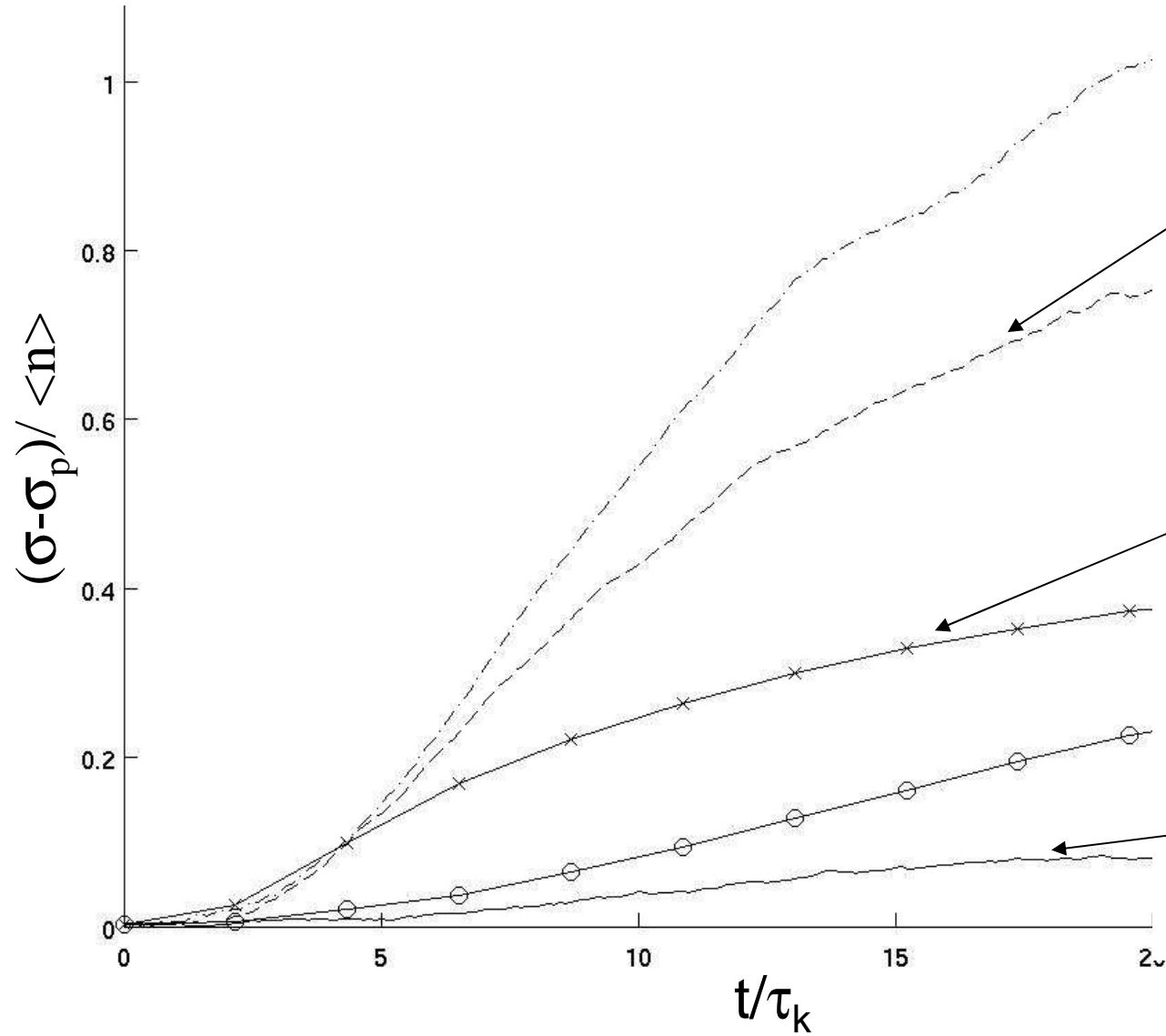
# Initial transient of accumulation



# Initial transient of accumulation

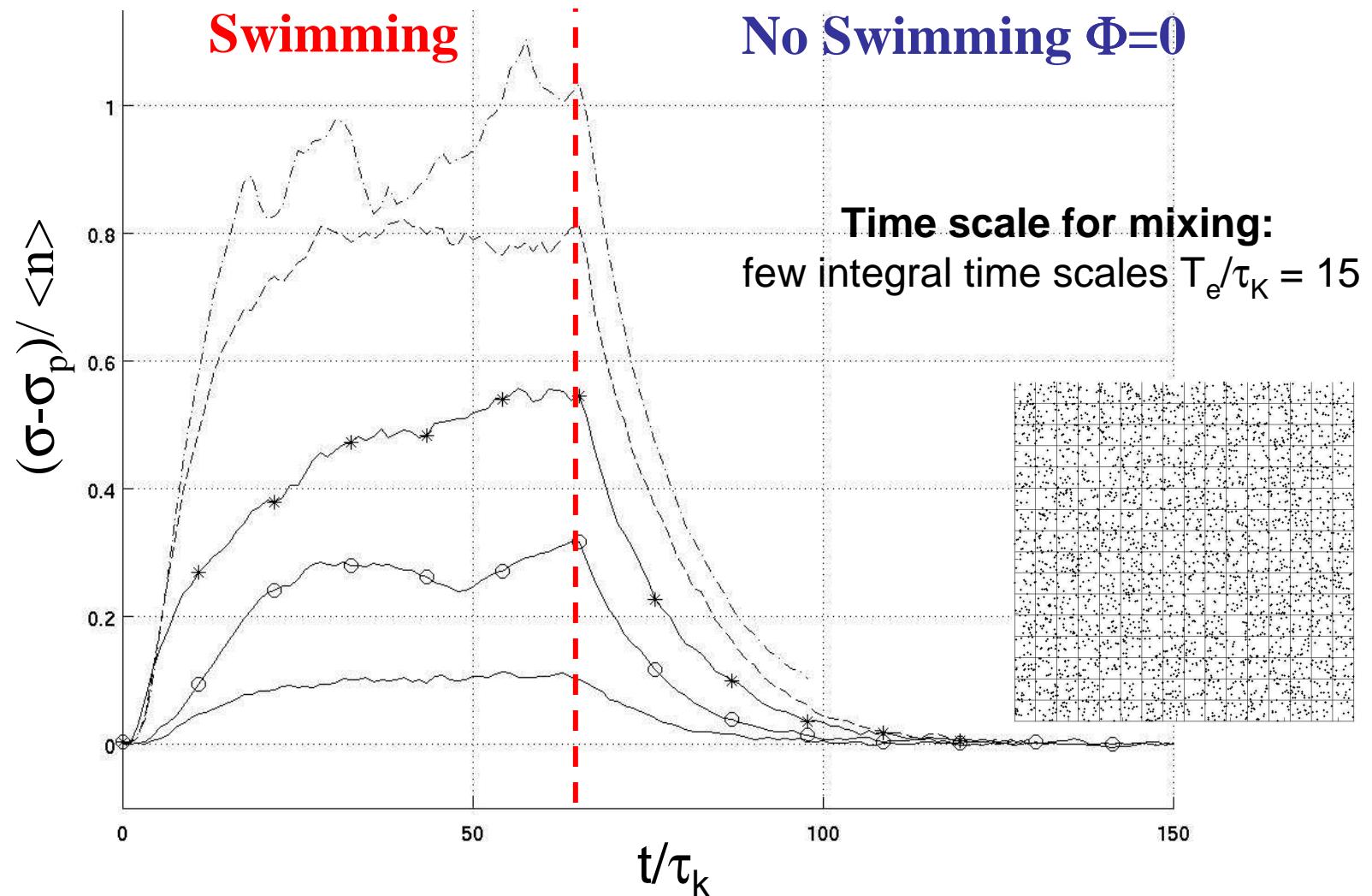


# Initial transient of accumulation



20

# Back to mixing

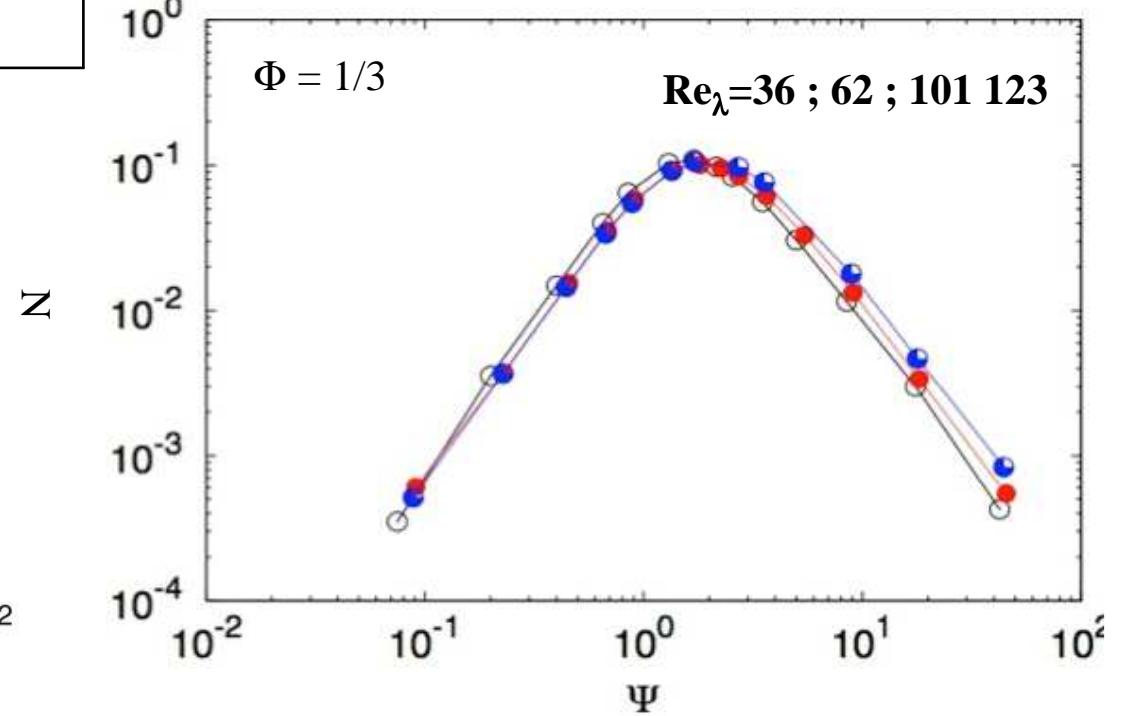
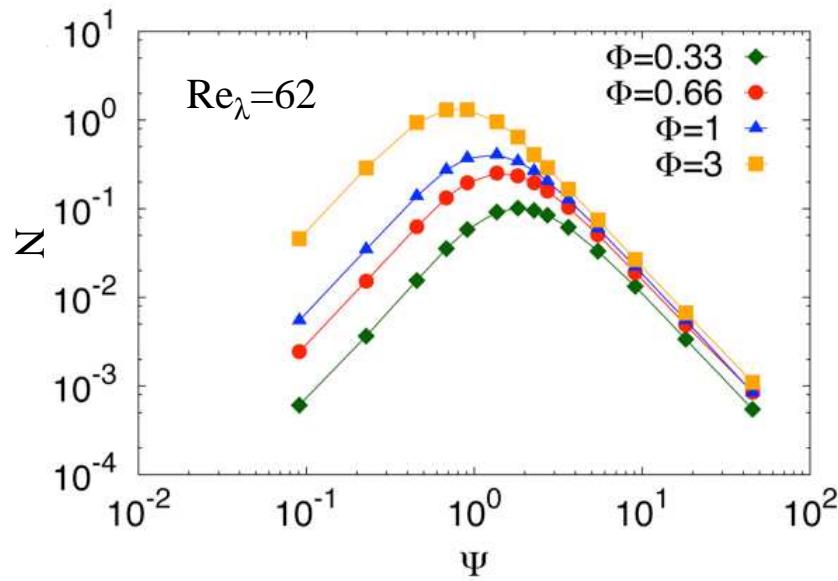


# Kolmogorov Scaling

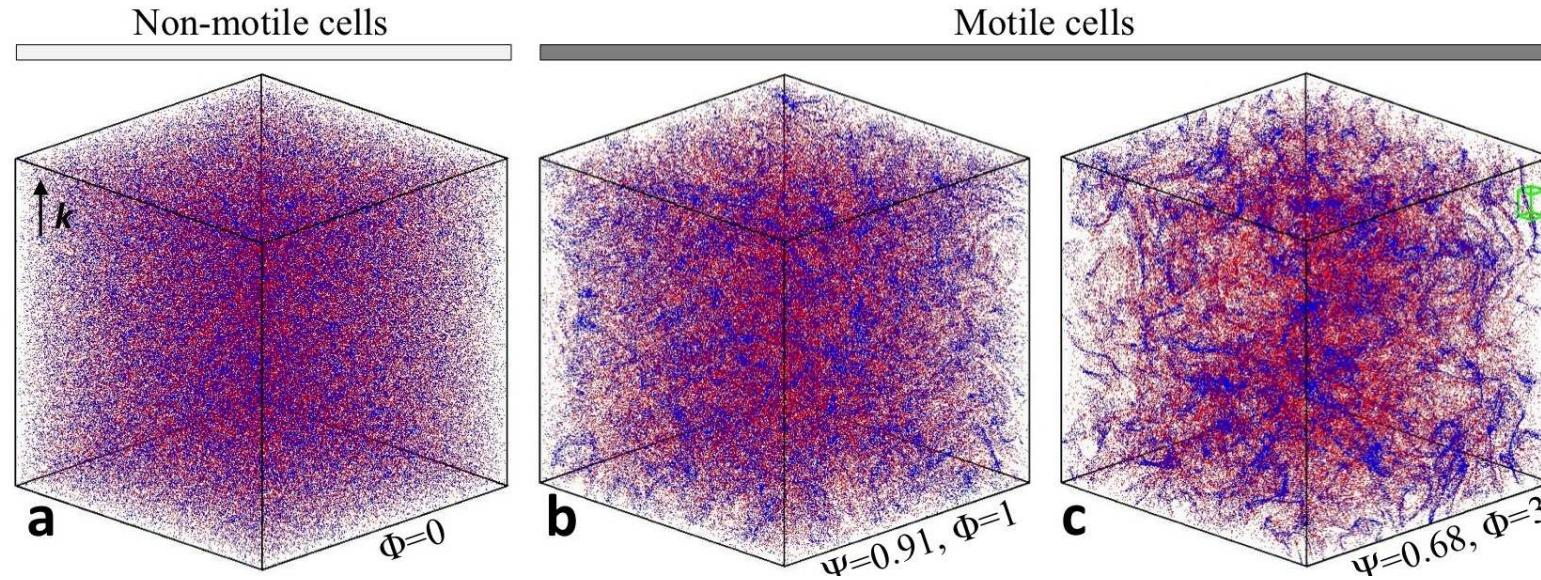
$Re_\lambda$	$M$	Number of cells	$\lambda/\eta_K$
36	$64^3$	300,000	11.7
62	$128^3$	300,000	15.7
101	$256^3$	2,400,000	19.5
123	$512^3$	3,200,000	21.7

$\Psi = B/\tau_K = B/\text{Kolmogorov time}$

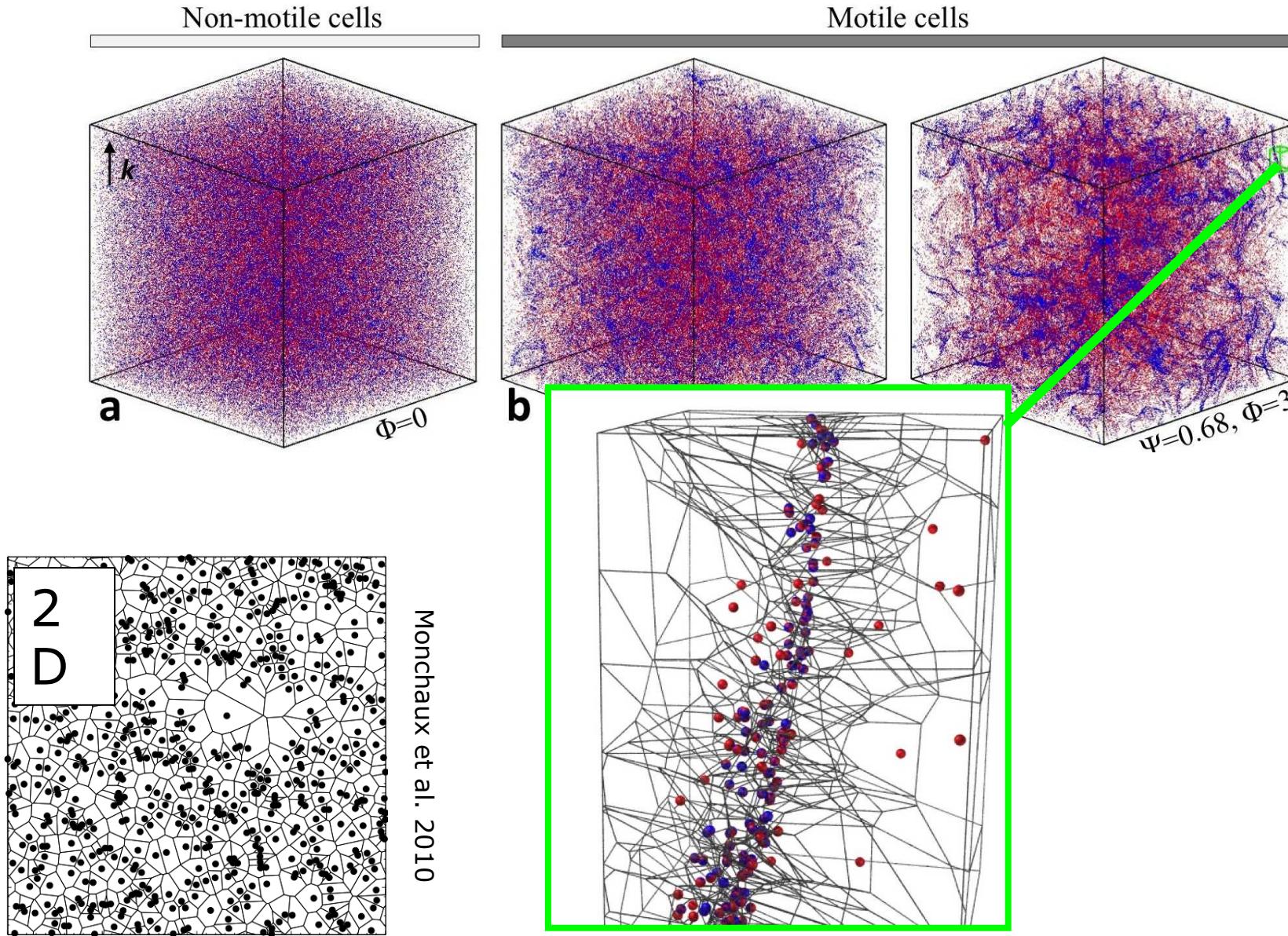
$\Phi = \frac{V_{\text{Swim}}}{v_K} = \frac{\text{Swimming velocity}}{\text{Kolmogorov velocity}}$



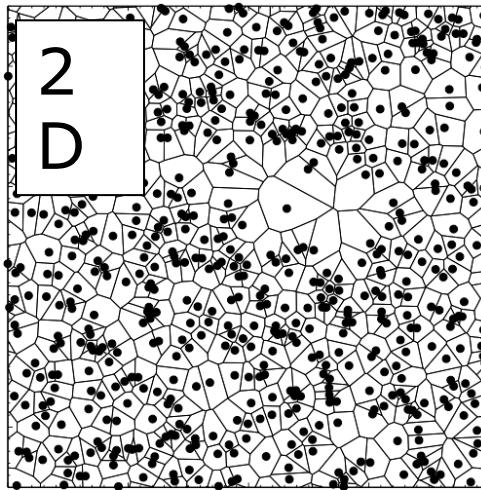
# Quantifying Patchiness with a Voronoi Tesselation



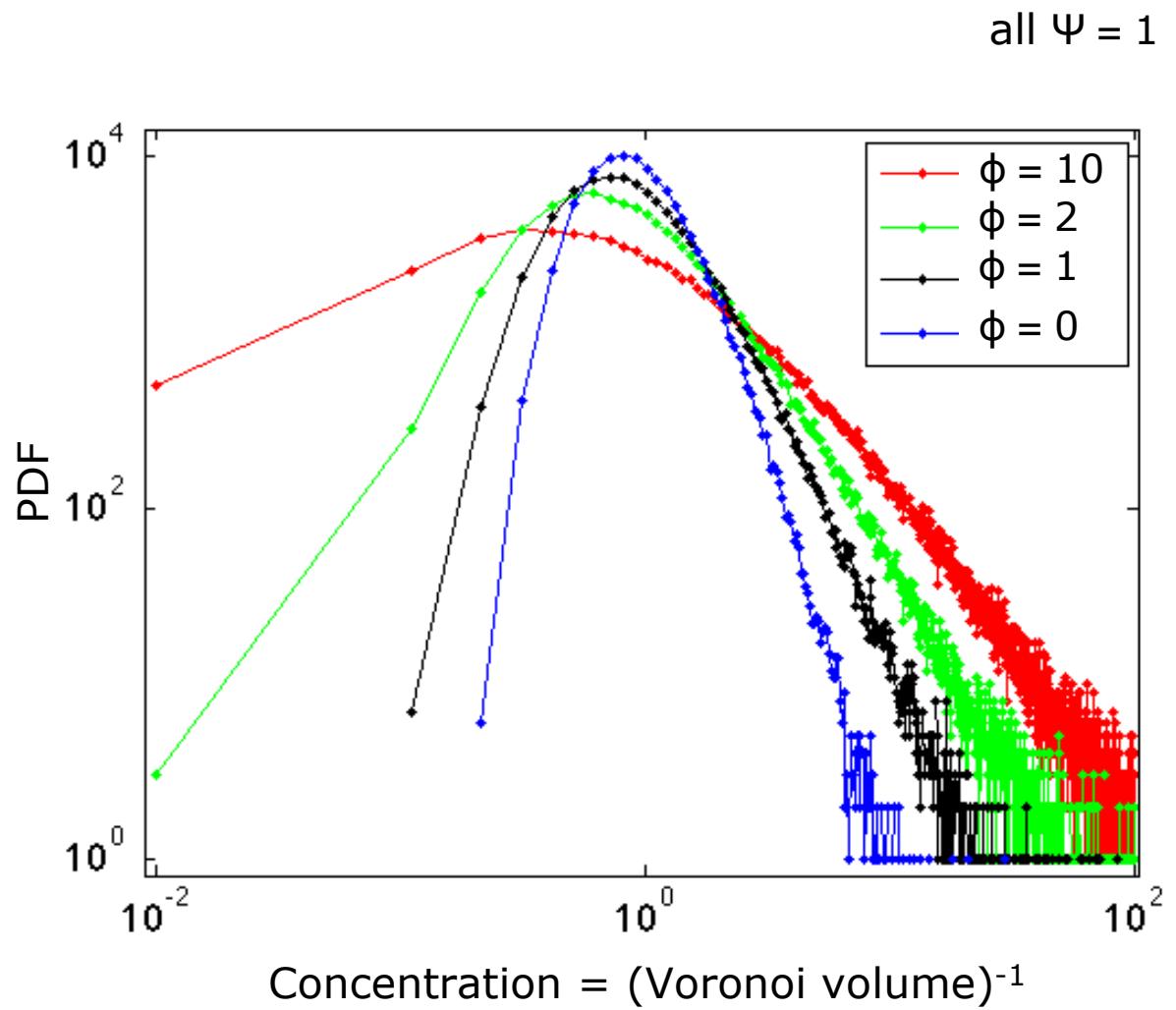
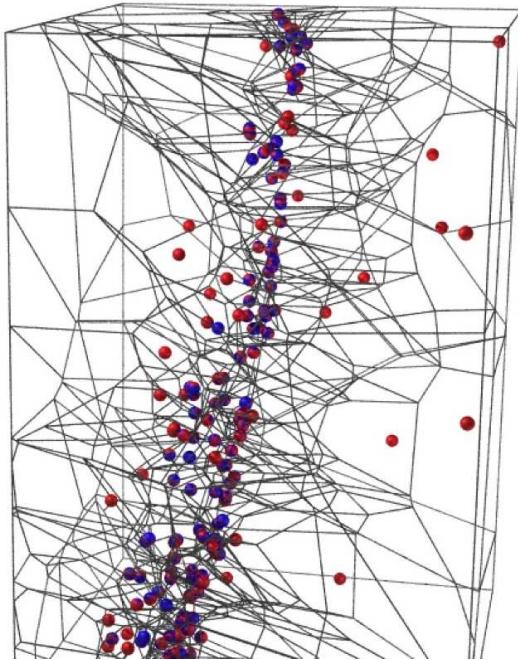
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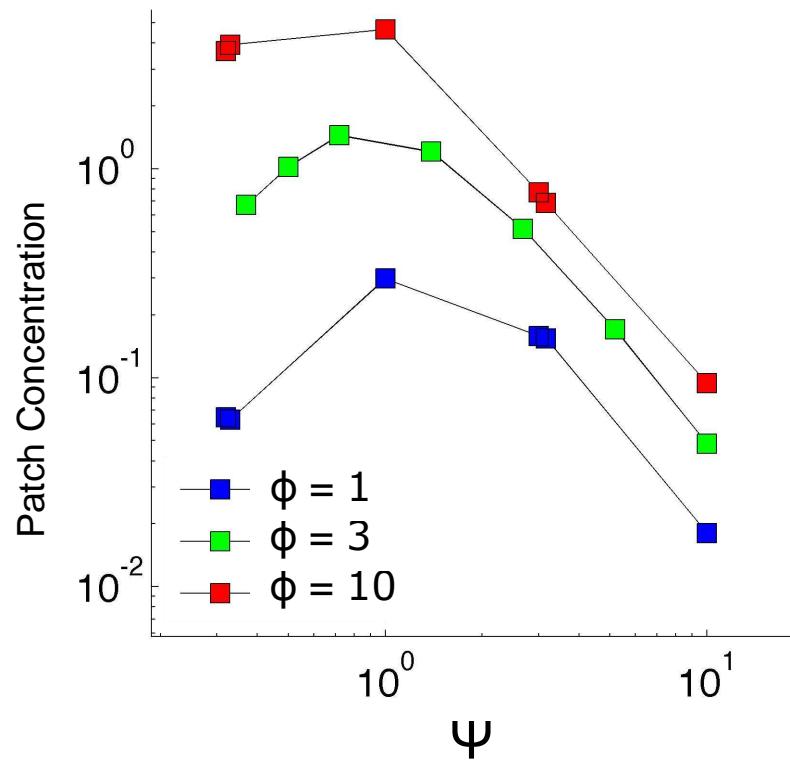
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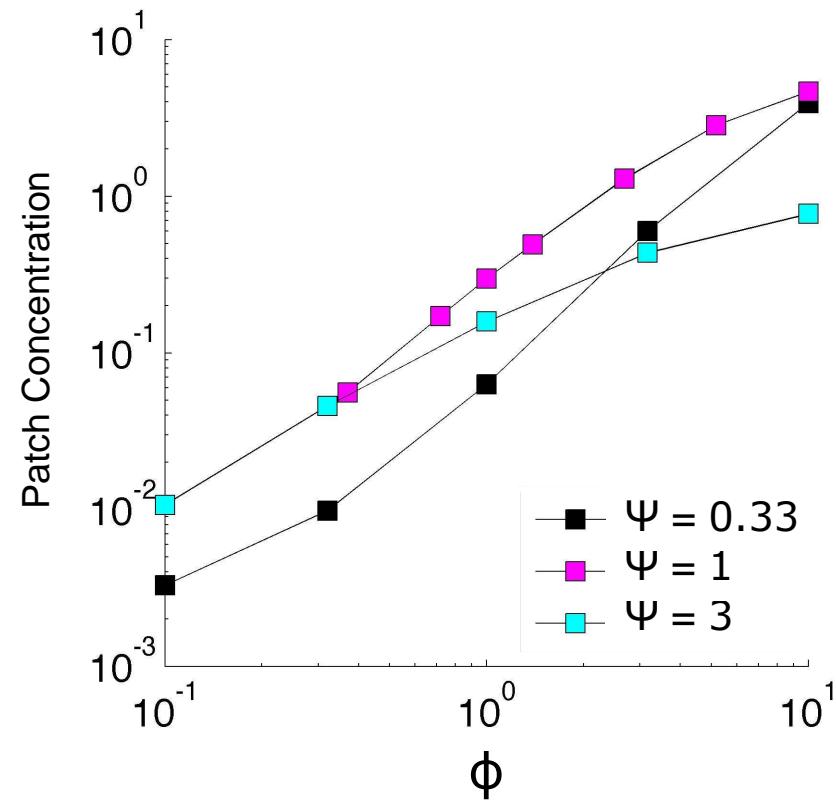
Monchaux et al. 2010



# Patchiness as a function of $\Psi$ and $\phi$



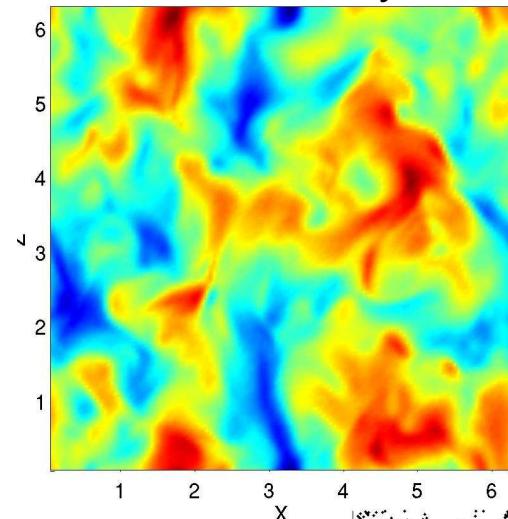
Patchiness most  
intense for 'neutral stability'



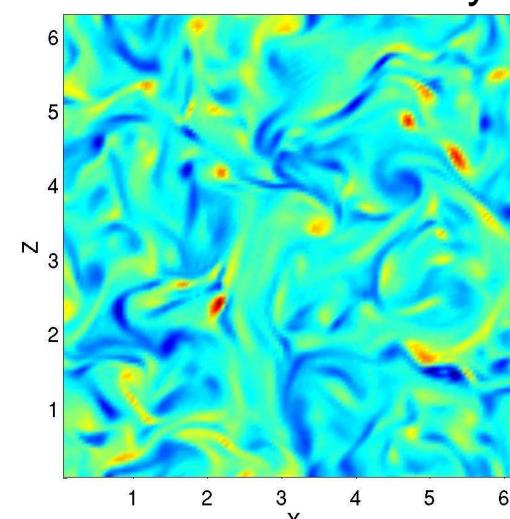
Increased swimming speed  
produces more intense patches

# Accumulation mechanism

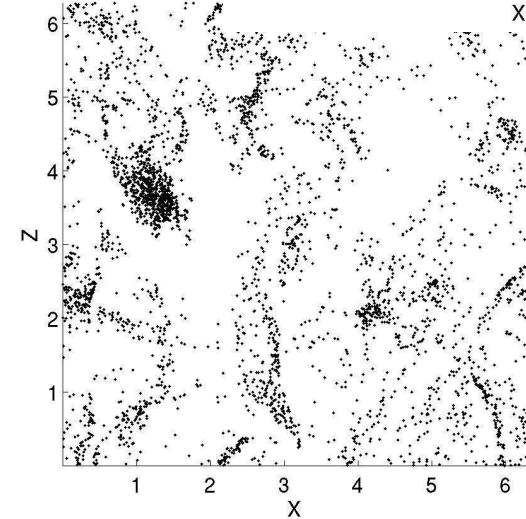
Vertical velocity  $w$



Horizontal vorticity



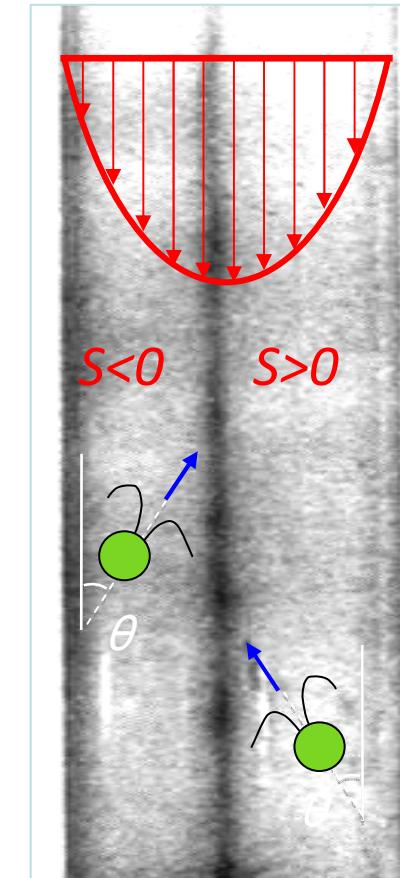
$$\Phi=10 ; \Psi=1$$



Average fluid velocity at cells position

$\Psi=1$ and $\Phi=$	0	1	2	10	20
$(\sigma - \sigma_p)/\langle n \rangle$	0	0.1	0.5	0.8	1.1
$\langle w _x \rangle / u'$	$10^{-4}$	-0.08	-0.15	-0.31	-0.74

$\downarrow g$



Focusing in  
downwelling  
flows  
(Kessler, 1985)

## More details and references

W. Durham, E. Climent, M. Barry, F. De Lillo, G. Boffetta, M. Cencini and R. Stocker  
*Nature Communications* (2013) under revision.

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W.M. Durham, E. Climent and R. Stocker. *Phys. Rev. Letters* – 106, 238102 (2011).  
“Gyrotaxis in a steady vortical flow”.

Durham, W. M., Kessler, J.O., and Stocker, R. - *Science*. 323 (2009).  
“Disruption of vertical motility by shear triggers formation of thin phytoplankton layers.”

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“Hydrodynamic focusing of motile algal cells.”

## Acknowledgements and funding

Regional HPC – Calmip (2011-2012).

BQR INPT International Mobility – 2008 – 2009  
MIT-France Seed Fund – 2010-2012